Columbia Engineering
The Fu Foundation School of Engineering and Applied Science

ELEN E3106/4106 Lecture 18

BJTs Part IV: Models and Secondary Effects

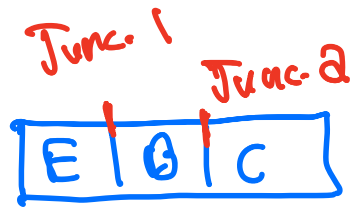
Outline

- Ebers-Moll model
- Drift Transistors
- Total Transit time & Kirk Effect
- Small-signal mode & capacitance
- Cut-off Frequency
- Charge control model

Assignments:
Reading: C. Hu §8.6-8.10
Homework 7 due Fri. Nov. 14th by 5pm

Recap on BJT types

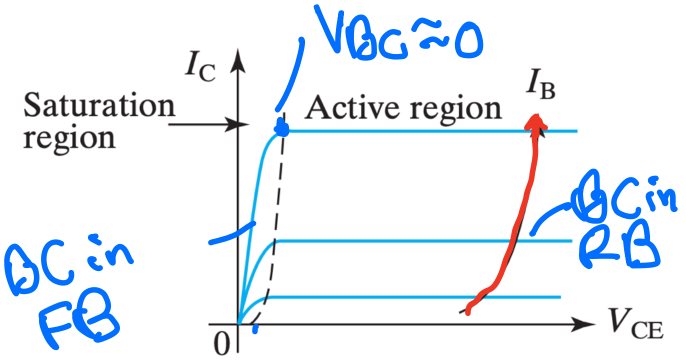
- It's important to remember that *forward biased* p-n junctions have a more positive voltage on the p-side relative to the n-side
- In *reverse bias*, the n-side is more positive relative to the p-side
- Generally, voltage subscripts are denoted p to n side (V_{pn})
  - For npn, the E and C are n-type and B is p-type so V_{pn} would be V_{BE} and V_{BC}
  - For pnp, the E and C are p-type and B is n-type V_{pn} would be V_{EB} and V_{CB}

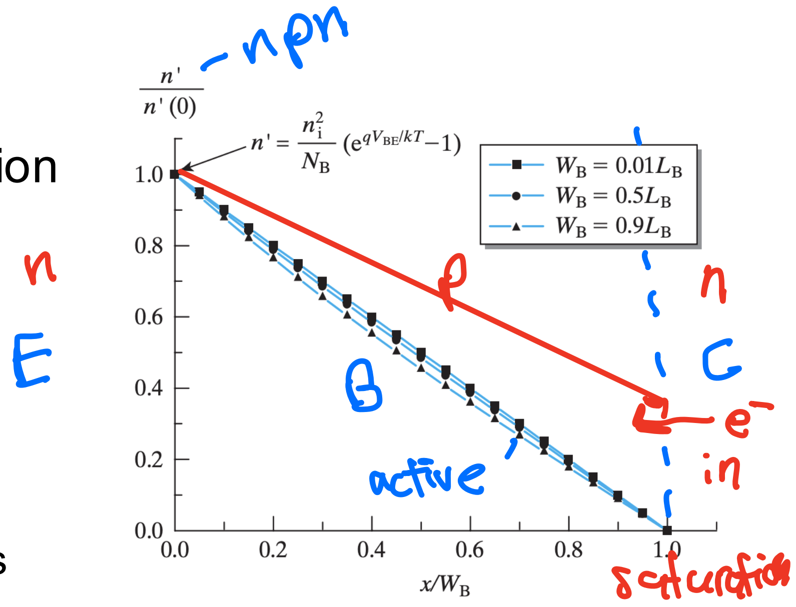


- Ex. in active/forward mode, the EB junction is forward biased an the CB is reverse biased *regardless of BJT type*
  - In npn, $V_{BE} > 0$ and $V_{BC} < 0$
  - In pnp, $V_{EB} > 0$ and $V_{CB} < 0$
- The total voltage from collector to emitter is just the sum of the voltages across the 2 junctions: $V_{CE} = V_{CB} + V_{BE}$
  - But we careful again with voltage polarity!
  - Ex. We have an npn in saturation mode (both junctions FB) with $V_{BE} = 0.8\ V, V_{BC} = 0.6\ V$, hence $V_{CE} = 0.8 - 0.6\ V = 0.2\ V$

A Note on Saturation and Linear Regimes

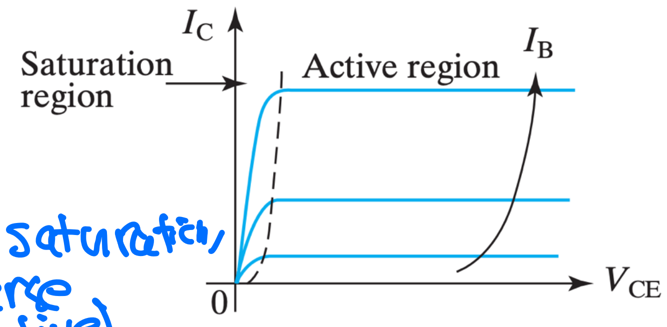
- Let's take another look at why I_C decreases as $V_{CE} \rightarrow 0$ in the saturation regime
  - Both junctions are FB. Increasing FB V_{BC} decreases V_{CE} for a given constant V_{BE} ($V_{CE} = V_{CB} + V_{BE}$)
  - In an npn, when BC is FB, the excess e- ($\Delta n_p = n'$) at x = W_B rises as the collector also injects e- into the base
  - $\frac{dn}{dx}$ is depressed, and therefore I_C is depressed
- The dashed line boundary between the transistor saturation and active modes represents where $V_{BC} =$ 0
  - E.g. the CB junction goes from FB to RB
- And why is I_C practically flat in active mode?
  - Recall that I_C is essentially independent of V_{CB}, as long as V_{CB} is RB
  - Instead, I_C depends on V_{BE}. So even as we increase RB V_{BC} and V_{CE} increases for a given V_{BE} ($V_{CE} = V_{CB} + V_{BE}$), I_C remains the same



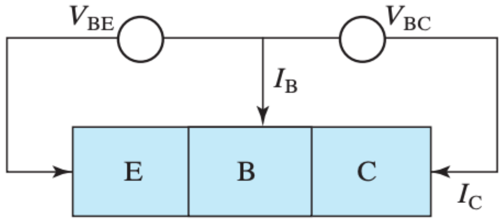


Sources: C. Hu *Modern Semiconductor Devices for Integrated Circuits*

Ebers-Moll Model



- A popular model (frequently used in SPICE circuit simulation) that mathematically describes the BJT in all 4 modes of operation (active/forward, saturation, cut-off, reverse active)
  - We will look at saturation and active modes



- Let's first assume that $V_{BC} = 0$ in an npn. Our current equations become:

$$I_C = I_S(e^{qV_{BE}/kT} - 1)$$

$$I_B = \frac{I_S}{\beta_F}(e^{qV_{BE}/kT} - 1)$$

* β_R is beta in reverse active mode

- Now let's assume the opposite ($V_{BE} = $ 0):
  - e- are injected from C -> B and flow to emitter

$$I_E = I_S(e^{qV_{BC}/kT} - 1)$$

$$I_B = \frac{I_S}{\beta_R}(e^{qV_{BC}/kT} - 1)$$

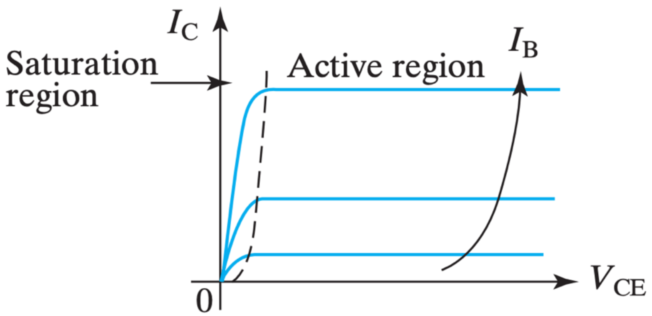
$$I_C = -I_E - I_B = -I_S\left(1 + \frac{1}{\beta_R}\right)(e^{qV_{BC}/kT} - 1)$$

Sources: C. Hu *Modern Semiconductor Devices for Integrated Circuits*

Ebers-Moll Model

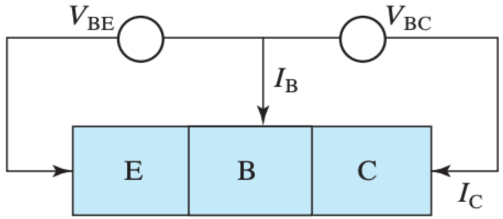
*large signal model

- When both $V_{BC}, V_{BE} \neq 0$, our eq's are superimposed, comprising the Ebers-Moll model:

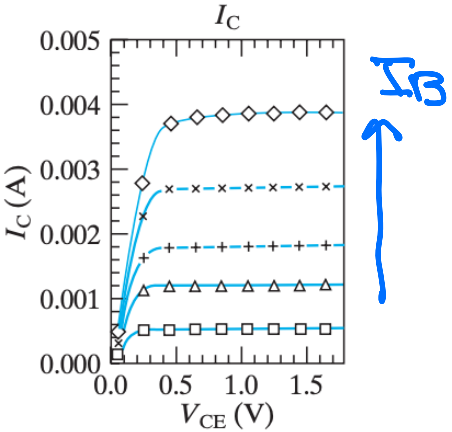


$$I_C = I_S(e^{qV_{BE}/kT} - 1) - I_S\left(1 + \frac{1}{\beta_R}\right)(e^{qV_{BC}/kT} - 1)$$

$$I_B = \frac{I_S}{\beta_F}(e^{qV_{BE}/kT} - 1) + \frac{I_S}{\beta_R}(e^{qV_{BC}/kT} - 1)$$



- The model (lines) agrees with experimentally measured data (symbols) in both the saturation and linear regimes. Example shown is for an SiGe-base HBT.



Sources: C. Hu *Modern Semiconductor Devices for Integrated Circuits*

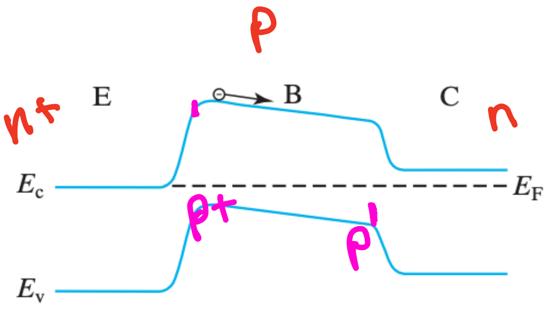
Drift Transistors: Built-In Base Fields

- How else can we reduce base transit time and improve speed?
  - Recall: we discussed compositionally grading the base to establish an electric field that aids the flow of carriers from emitter to collector

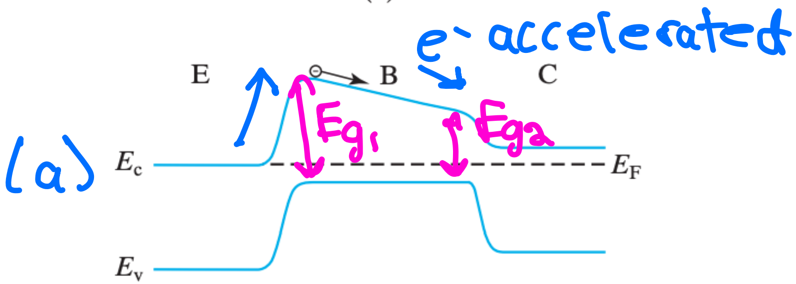
(a)

(b)

- Alternatively, we could create a built-in base field by varying the doping concentration, N_B, through the base
- The varying N_B creates $\frac{dE_v}{dx}$ and $\frac{dE_C}{dx}$, and the field
- In this case, the base transit time is NOT reliant on diffusion ($\tau_t \neq \frac{W_B^2}{2D_B}$) but instead reliant on drift!
- We can achieve $\tau_t << \frac{W_B^2}{2D_B}$
- These BJTs are called drift transistor



(a)



(b)

$E_{g1} > E_{g2}$

Sources: C. Hu *Modern Semiconductor Devices for Integrated Circuits*

Emitter-to-Collector Transit Time

- What's the total time it takes for carries to cross from E -> C?
  - τ_F, total forward transit time (Units: s) $\tau_F = \tau_E + \tau_t + \tau_C$
    base transit time
- What portion of τ_F is made up by the base transit time, τ_t?

  About half

- What else contributes to transit (storage) time?

  emitter, collector widths (SCR and QNRs)

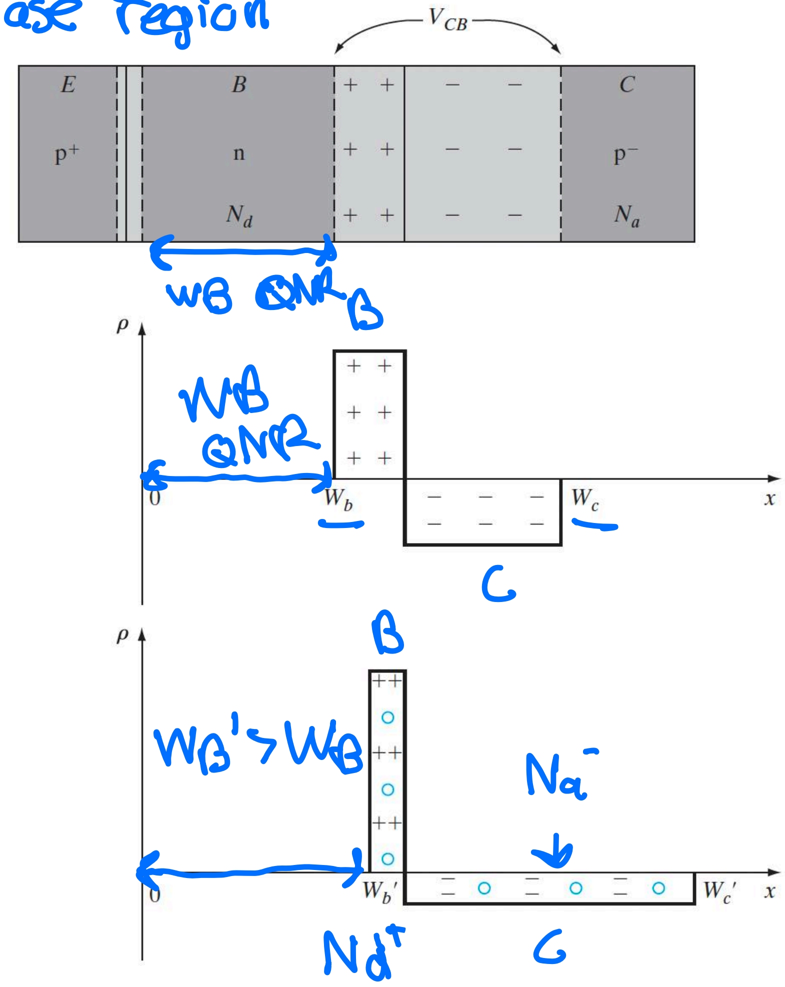
- As I_C increases, τ_F increases. Why? Ex. npn
  - The dopant density (N_B = p) is insufficient to support the collector current, even when carriers move at saturation velocity (v_{sat})

Sources: C. Hu *Modern Semiconductor Devices for Integrated Circuits*

Kirk Effect

Recall: Early effective → narrowing of the base region

- Effective widening of the base region at high I_C!
- Also causes current gain drop at high collector currents!
  (β will drop!)
- Reverse biased CB junction has a build up of mobile carriers due to strong collector current
- Let's take a pnp BJT. The injected h+ mobile charges add to the fixed N_d^+ on the base side of the CB depletion region, and subtract from the fixed N_a^- on the collector side
- Fewer uncompensated donors (and thus a smaller depletion width) are needed to maintain the reverse voltage
- The base width increases from W_B to W_B'



Sources: C. Hu *Modern Semiconductor Devices for Integrated Circuits*

Kirk Effect

- Remember: the depletion region extends more into the more lightly doped (collector) side
- This is equivalent to moving the base-collector junction deeper into the collector
- Drops the current gain due to an increase of the base transit time!
- With even higher current levels and base widening, what effect can occur in the CB junction? punchthrough of collector

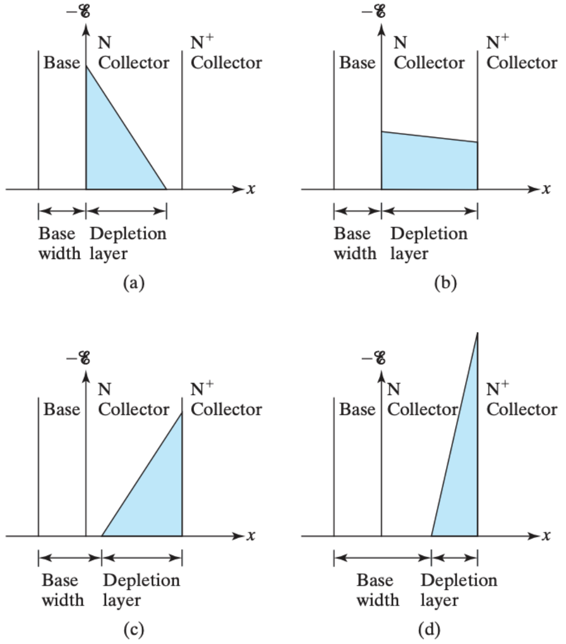
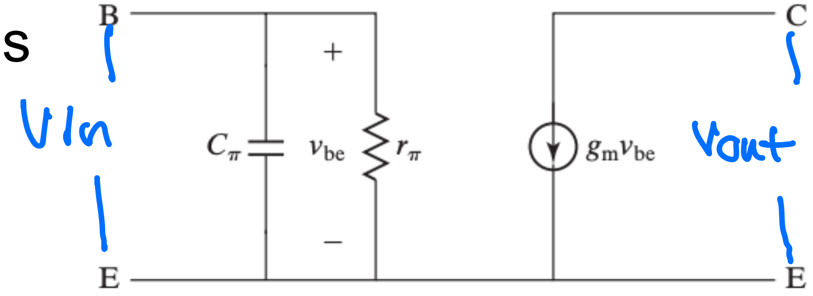


FIGURE 8–18 **Electric field** ℰ(x), location of the depletion layer, and base width at (a) low I_C such as 0.1 mA/μm^2 in Fig. 8–17; (b) larger I_C; (c) even larger I_C (such as 1 mA/μm^2) and base widening is evident; and (d) very large I_C with severe base widening.

Sources: C. Hu *Modern Semiconductor Devices for Integrated Circuits*

Small-Signal Model (Hybrid π models)

- Small-signal models apply when we have an AC signal that is small relative to the DC bias currents and voltages.
  - Useful for analog circuits!
  - These models are AC equivalent circuits where nonlinear circuit elements are replaced by linear elements whose values are approximated to the 1st order
- Ex: Let's say we have a small input signal (v_{BE}~10 mV sinusoidal), superimposed on a DC bias
- Which configuration is this? Common emitter



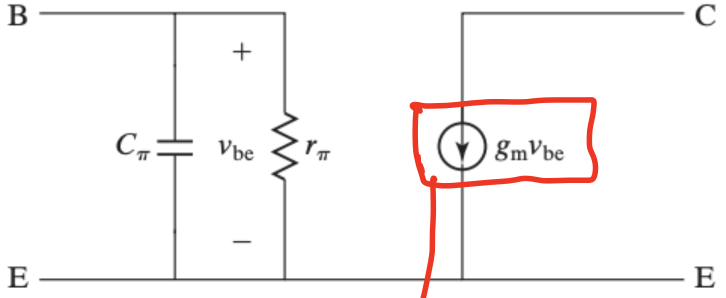
Sources: C. Hu *Modern Semiconductor Devices for Integrated Circuits*

Small-Signal Model and Transconductance

only for forward active mode

- Assuming V_{BE} is not near zero, we can neglect the "1" term:

$$I_C = I_S(e^{\frac{qV_{EB}}{kT}} - \cancel{1})$$



current source

$v_{be} \to$ AC signal

- The BJT *transconductance*, g_m, is the change in current with the change in voltage:

$$\boxed{g_m \equiv \frac{dI_C}{dV_{BE}}} = \frac{d}{dV_{BE}}(I_S e^{qV_{BE}/kT})$$

$$= \frac{q}{kT} I_S e^{qV_{BE}/kT} = I_C / \frac{kT}{q}$$

$$\boxed{g_m = I_C / \frac{kT}{q}} \leftarrow \text{thermal voltage}$$

- So what's g_m at 300 K? $g_m = I_C / 26\text{ mV} = I_C / 0.026$
- In general, we want to maximize g_m

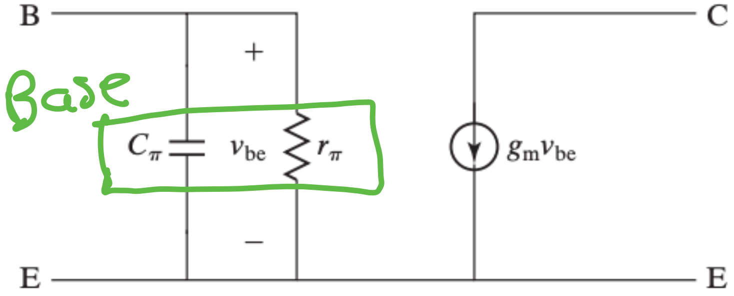
Sources: C. Hu *Modern Semiconductor Devices for Integrated Circuits*

Small-Signal Model and Charge-Storage Capacitance

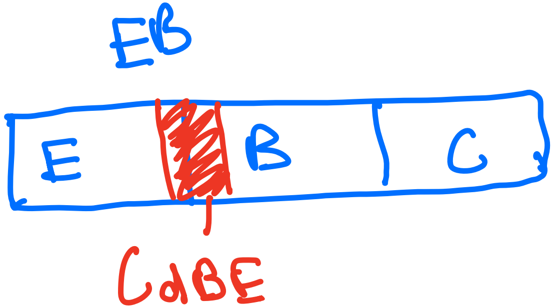
- The input (base) appears as a parallel RC circuit with

$$\frac{1}{r_\pi} = \frac{dI_B}{dV_{BE}} = \frac{1}{\beta_F}\frac{dI_C}{dV_{BE}} = \frac{g_m}{\beta_F}$$

$$r_\pi = \beta_F / g_m$$



- Q_F is the excess carrier charge stored in BJT
  - If we have $Q_F = 1\ pC$, we have 1pC of h+ and 1pC of e-
  - In npn, excess h+ charge is supplied by I_B



- So, the base presents this capacitance to the input:

$$C_\pi = \frac{dQ_F}{dV_{BE}} = \frac{d}{dV_{BE}}\tau_F I_C = \tau_F g_m$$

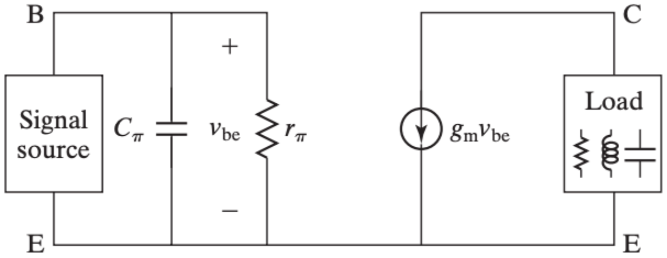
- This *charge-storage capacitance* (diffusion capacitance) must also take into account the EB junc. depletion-layer charge, C_{dBE}

$$C_\pi = \tau_F g_m + C_{dBE}$$

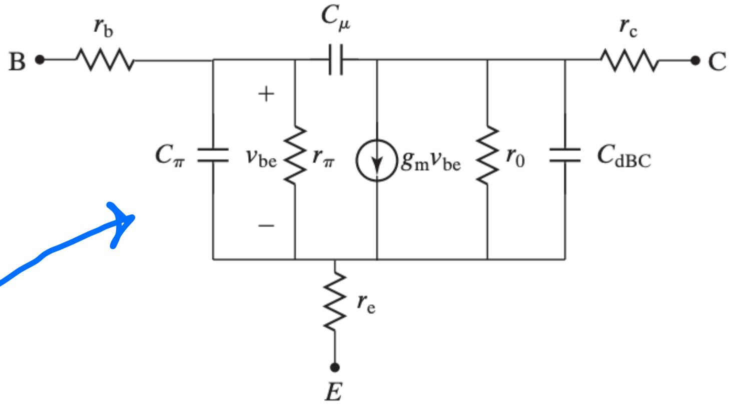
Sources: C. Hu *Modern Semiconductor Devices for Integrated Circuits*

Small-Signal Model

- What's the significance of the small-signal modal?
- Now we can analyze circuits with arbitrary signal-source and load impedence network by hand!
- This simple small-signal model can be solved to find g_m, r_π, C_π
- Or, SPICE circuit simulations can be used with a more accurate small-signal model (bottom)
  - Includes effects of base width modulation, etc.



(a)



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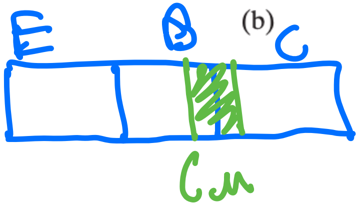
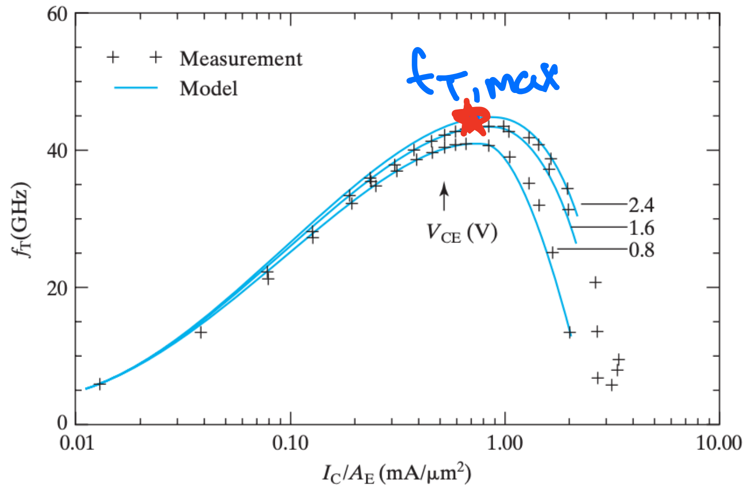


Sources: C. Hu *Modern Semiconductor Devices for Integrated Circuits*

Cut-Off Frequency

- f_T is the *cut-off frequency,* commonly used parameter to compare the speed of transistors. A more complete small-signal model yields:

$$f_T = \frac{1}{2\pi[\tau_F + (C_{dBE} + C_{dBC})kT/(qI_C) + C_{dBC}(r_e + r_c)]}$$



- Why? It can be shown that the frequency of *unity power gain* (β falls to 1, max. oscillation frequency) is given,

$$f_{max} = \left(\frac{f_T}{8\pi r_b C_{dBC}}\right)^{1/2}$$

- f_T rises with increasing I_C due to increasing g_m
- At very high I_C, τ_F increases due to base widening and f_T decreases
- So where do we bias BJT the best high-frequency performance?

Sources: C. Hu *Modern Semiconductor Devices for Integrated Circuits*

Charge Control Model

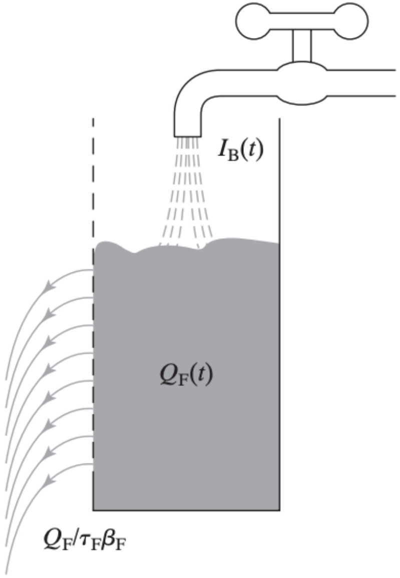
- What if the input signal is large? We can't use small signal model. What $I_C(t)$ is produced by a time-varying $I_B(t)$?
- Even if the charge storage varies with time, the relation still holds true:

$$\boxed{I_C(t) = Q_F(t)/\tau_F}$$

- This suggest I_C is controlled by Q_F, hence the name *charge control model.* In DC, $I_B \propto Q_F$:

$$I_B = I_C/\beta_F = Q_F/\tau_F\beta_F$$

- AKA In order to sustain a constant excess hole charge, h+ must be supplied to the transistor through I_B to replenish the h+ lost to recombination.



- Analogy would be filling a leaky bucket from a faucet, where the amount of water $Q_F(t)$ rises or falls at the rate of supply $I_B(t)$ minus leakage $Q_F/(\tau_F\beta_F)$

$$\boxed{\frac{dQ_F}{dt} = I_B(t) - \frac{Q_F}{\tau_F\beta_F}}$$

Sources: C. Hu *Modern Semiconductor Devices for Integrated Circuits*